বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2022

(Under CBCS Pattern)

Semester - II

Subject : MATHEMATICS

Paper : C 3 - T

[REAL ANALYSIS]

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

1. Answer any ***five*** questions : 2×5=10

(a) Define least upper bound of a bounded set and obtain it for the set

$A=\left\{\frac{1}{2},\frac{2}{3},\frac{3}{4},......\frac{n}{n+1},.....\right\}$

(b) Define point of accumulation of a set and find all the points of accumulation of the set $E=\left\{\frac{1}{m}+\frac{1}{n}/\,m,n=1,2,3,.....\right\}$.

(c) Prove that if A and B are two closed sets, then $A\cup B$ and $A\cap B$ are both closed sets.

P.T.O.

(d) Show that $\left\{\frac{3n+1}{n+2}\right\}$ is a bounded sequence.

(e) Prove that every convergent sequence is bounded. Is the converse true? Justify.

(f) Prove that the Series $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ converges.

(g) If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then prove that $\lim_{n\to\infty} a_n = 0$.

(h) Define compact set with an example.

2. Answer any ***four*** questions : 5×4=20

(a) Define countability of a set. Show that the set of all real numbers is not countable.

(b) State and prove Archimedean property of real numbers.

(c) If a set S is open, then prove that its complement is a closed set. Is the converse true? Justify.

(d) Define Cauchy Sequence. Prove that the sequence $\{n^2\}$ is not a Cauchy Sequence.

(e) Prove that every bounded sequence has a convergent subsequence.

(f) Prove that $1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+.......$ converges.

3. Answer any ***three*** questions : 10×3=30

(a) What do you mean by convergence, absolute convergence and conditional convergence of a series of real numbers? Prove that absolutely convergence imply convergence. Classify as to divergent, conditionally convergent or absolutely of the following series :

(i) $1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+.....$

(ii) $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+.....$

(iii) $\frac{1}{2}-\frac{2}{3}+\frac{3}{4}-\frac{4}{5}+.....$ 3+1+6

(b) If a sequence $\{x_n\}$ of real numbers is monotonic increasing and bounded above, then prove that it converges to its exact upper bound. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is monotonic increasing and bounded above. 5+5

(c) (i) State and prove Bolzano-Weierstrass theorem for sequences.

(ii) Using Cauchy's general principle of convergence prove that $\{x_n\}$, where $x_n = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....+(-1)^{n-1}\cdot\frac{1}{n}$, is a convergent sequence. 5+5

(d) (i) State and prove Heine-Borel theorem. Give an illustration which justify Heine-Borel theorem.

(ii) State and prove density property of real numbers. 4+3+3

(e) (i) Examine if the following series converge :

(i) $\sum_{n=1}^{\infty}\frac{n+1}{10^{10}(n+2)}$ (ii) $\sum_{n=1}^{\infty}\frac{n+1}{n+2}$ (iii) $\sum_{n=1}^{\infty}\log\left(1+\frac{1}{n}\right)$

(ii) Given $0 < x_1 < x_2$. If each $x_n = \frac{x_{n-1}+x_{n-2}}{2}$,

then prove that $\{x_n\} \to \frac{1}{3}(x_1+2x_2)$ as $n \to \infty$. 6+4